DISCUSSION PAPER

No 09

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September 2010

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UNIVERSITÄT DÜSSELDORF

The Euro Area Interbank Market and the Liquidity Management of the Eurosystem in the Financial Crisis[*]

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September 2010[§]

Abstract

This paper develops a theoretical model which explains several stylized facts observed in the euro area interbank market after the collapse of Lehman Brothers in 2008. The model shows that if costs of participating in the interbank market are high, the central bank assumes an intermediary function between liquidity surplus banks and liquidity deficit banks and thereby replaces the interbank market. From a policy perspective, we argue that possible measures of the Eurosystem to reactivate the interbank market may conflict, inter alia, with monetary policy aims.

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1 Introduction

The worldwide financial crisis, which broke out in August 2007, triggered severe turbulence in the euro area money markets. Particularly in the aftermath of the collapse of Lehman Brothers in September 2008, several previously unseen developments could be observed. Transactions in the interbank markets fell dramatically and the interest rate for overnight interbank lending, which is usually slightly above the Eurosystem's[1] key policy rate, declined significantly below this rate. At the same time, aggregate borrowing of euro area commercial banks from the Eurosystem but also their use of the Eurosystem's deposit facility rose sharply. The aim of this paper is to theoretically explain these stylized facts and to discuss some policy implications.

We present a theoretical model which shows that the developments in the euro area during the financial crisis can be explained by high costs of participating in the interbank market. Participation in this market was particularly costly during the crisis because the crisis implied high bank asset losses combined with a high degree of uncertainty in how far individual banks were affected. This led to higher costs for potential lenders as well as for potential borrowers. Lenders had to screen more thoroughly potential borrowers, the latter had to put more effort in signalling their creditworthiness. Credit lines were cut so that on both market sides there were banks which had to find a higher number of suitable transaction partners. Besides, banks in need of liquidity feared that borrowing large amounts in the interbank market might damage their reputation. All these developments made participation in the unsecured overnight interbank market relatively costly during the crisis. For some deficit banks participation costs even became prohibitive. They had no longer access to this market as lenders are unwilling to expose themselves to any counterparty credit risk.[2]

For explaining the above sketched stylized facts, our theoretical analysis suggests that the increased costs of participating in the interbank market implied that there were deficit banks finding it more attractive to borrow directly from the central bank rather than in the interbank market and other deficit banks that were forced to borrow from the central bank since they were blocked from the interbank market. This led to a significant increase in the demand for central bank credit. Since the ECB fully satisfied this increased demand, there

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[1]The term "Eurosystem" stands for the institution which is responsible for monetary policy in the euro area, namely the ECB and the national central banks in the euro area. For the sake of simplicity, the terms "ECB" and "Eurosystem" are used interchangeably throughout this paper.

[2]This is because even for very small default risks, the potential losses would outweigh the returns of overnight lending. See footnote 23 for a numerical example.

was excess liquidity in the banking sector which put downward pressure on the interbank market rate and which induced banks with a liquidity surplus to place their excess liquidity in the deposit facility offered by the Eurosystem. Often it is argued that the intensified use of the deposit facility reflects excess reserves held for precautionary motives.[3] Our theoretical analysis offers a further explanation. We argue that as a consequence of large participation costs, the ECB replaced the interbank market by assuming an intermediary function between surplus and deficit banks.[4]

The aim of the Eurosystem is to reverse its intermediary function and to reactivate interbank market transactions (Smaghi, 2008). According to our model, the obvious way to achieve this aim is to reduce interbank market participation costs. However, this cannot be accomplished by the Eurosystem since it cannot reduce informational problems and uncertainties about the soundness of financial institutions. Our model shows that a central bank can reactivate interbank market transactions by making its transactions with the banking sector less attractive, which can be done, for example, by increasing requirements for collateral or by decreasing the rate on the deposit facility. However, as long as transactions in the interbank market are associated with high costs, these activities will increase the banks' liquidity costs. In an economic and financial crisis, this increase usually conflicts with a central bank's aims from a monetary policy and financial stability perspective. Therefore, we suggest that the Eurosystem should make its transactions with the banking sector less attractive gradually over time, as informational problems and uncertainties become less and less significant.[5]

The related literature on the liquidity management of credit institutions, central bank activities and the consequences for the interbank market can be divided into three groups. The first group focusses on the liquidity management of U.S. banks, the Federal Reserve System and the U.S. federal funds market before the financial crisis.[6] The second group

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[3]See, for example, Trichet (2009a). For detailed analyses concerning precautionary hoarding of reserves during the financial crisis in the US and UK, see Ashcraft, McAndrews, and Skeie (2009) and Acharya and Merrouche (2009).

[4]Concerning this intermediary function see also European Central Bank (2009c).

[5]By the end of 2009, the Eurosystem started this kind of phasing-out policy. However, in May 2010, this process was suspended as new tensions in the financial markets arose (for details see González-Páramo, 2010).

[6]See, for example, Ho and Saunders (1985), Hamilton (1996), Clouse and Dow (1999, 2002), Furfine (2000), and Bartolini, Bertola, and Prati (2001, 2002).

concentrates on the euro area in the pre-crisis period.[7] Our paper belongs to the third group, which analyzes the credit institutions' and central banks' liquidity management during the financial crisis which started in August 2007. Allen, Carletti, and Gale (2009) discuss central bank measures to reduce the volatility of the interbank market rate. Eisenschmidt, Hirsch, and Linzert (2009) empirically analyze the banks' bidding behaviour in the ECB's main refinancing operations during the financial crisis until October 2008. Cassola and Huetl (2010) also look at the first phase of the financial crisis (July 2007 until August 2008). They investigate the consequences of the crisis on the rate and the transaction volume on the overnight interbank market. Eisenschmidt and Tapking (2009) analyze the evolution of liquidity risk premia in unsecured interbank markets. Bruche and Suarez (2010) and Heider, Hoerova, and Holthausen (2009) show that counterparty risk can lead to a decline in the transaction volume in the interbank market. Lenza, Pill, and Reichlin (2010) describe in detail the conduct monetary policy of the ECB, the Fed and the Bank of England. Borio and Disyatat (2009) classify the different unconventional measures undertaken by the central bank during the crisis and discuss their consequences for the transmission channel of monetary policy and the challenges these measures raise for policy makers, for example with regard to the relationship to the fiscal policy and the independency of the central bank. Our paper aims at providing a simultaneous explanation for the strongly increased demand for central bank liquidity, the significant use of the deposit facility, the decrease of the interbank market transaction volume, and the systematic decrease of the interbank market rate below the key policy rate in the euro area. Furthermore, respective policy implications are drawn.

The rest of the paper is organized as follows. In section 2, we present the institutional background. Section 3 describes the stylized facts to be explained by our theoretical model which is presented in section 4. In section 5, we discuss the results and the policy implications. The last section summarizes the paper.

2   Institutional Background

Deposits that banks hold on their accounts with the central bank plus the currency they hold are the *reserves* of the banking sector. In the euro area, the banking sector's needs for reserves arise from minimum reserve requirements and autonomous liquidity factors,

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[7]See, for example, Breitung and Nautz (2001), Ayuso and Repullo (2001, 2003), Ewerhart (2002), Bindseil (2002), Nautz and Oechssler (2003, 2006), Neyer and Wiemers (2004), Beaupain and Durré (2008) and Neyer (2009).

as banknotes in circulation. Needs for reserves can only be satisfied by the Eurosystem. It has monopoly power over the creation of reserves. This allows the Eurosystem to steer the interest rate in the interbank market for reserves, which is its operating target, by assessing the needs and providing or absorbing the appropriate amount of liquidity.

Important instruments for providing/absorbing reserves are the main refinancing operations (MROs), longer-term refinancing operations, fine-tuning operations and two standing facilities. The MROs are credit operations. They have a maturity of one week and are conducted weekly as either a fixed or a variable rate tender. For each MRO, the ECB calculates a benchmark allotment, which reflects the banking sector's liquidity needs during the maturity of the MRO if the reserve requirements are fulfilled smoothly over the reserve maintenance period.[8] In "normal" times, bids in the MRO will be rationed if total bids exceed the benchmark allotment. A further source of reserves for the banking sector are longer-term refinancing operations. In "normal" times they are conducted once a month and have a maturity of three months. Fine-tuning operations are non-standardized instruments to provide or absorb liquidity. Concerning the two standing facilities one has to distinguish between a credit facility and a deposit facility. Both have an overnight maturity. On the initiative of the banks, the credit facility provides liquidity, whereas the deposit facility absorbs liquidity. The interest rates on these facilities usually form a symmetric corridor around the MRO-rate. Credit operations with the Eurosystem have to be based on adequate collateral. Assets eligible as collateral must fulfil certain criteria defined by the Eurosystem.[9]

During the financial crisis, which broke out in August 2007, times were no longer "normal" and the ECB adopted a couple of non-standard-measures comprising the following six building blocks.[10] (1) The Eurosystem fully satisfied the banks' demand for liquidity although it exceeded the Eurosystem's benchmark allotment by far. (2) The list of assets eligible for use as collateral was expanded. (3) The range of maturities of longer-term refinancing operations was expanded up to one year. (4) The Eurosystem provided liquidity

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[8]The definition of the benchmark allotment reveals that although a single bank may fulfil its reserve requirements unevenly over the four-week reserve maintenance period (for fulfilling its reserve requirements, a credit institution can make use of averaging positions over the reserve maintenance period), the Eurosystem aims on aggregate a smooth fulfilment.

[9]For a detailed description of the Eurosystem's monetary policy instruments including its minimum reserve system and for information on the collateral framework see European Central Bank (2008). For a detailed description of the Eurosystem's liquidity management see European Central Bank (2002).

[10]For a brief survey see, for example, Trichet (2009b). For a detailed description of the implementation of monetary policy by the Eurosystem in response to the financial crisis see European Central Bank (2009c). Details on the Securities Market Programme can be found in González-Páramo (2010).

in foreign currencies. (5) The Eurosystem started to purchase euro-denominated covered bonds. (6) In May 2010, a Securities Market Programme was introduced which allows the ECB to intervene in public and private securities markets.

In the interbank market for reserves, banks exchange deposits they hold on their accounts with the Eurosystem. This market thus reallocates the reserves originally provided by the central bank. One reason for this reallocation is that usually, the shortest frequency by which the Eurosystem provides reserves to the banking sector is one week, namely through its MROs. While the needs for reserves of the banking sector as a whole may not change significantly within one week, the needs for reserves of individual banks usually fluctuate daily. These fluctuations result from cash withdrawals and cash deposits by the banks' customers and from bank transfer payments.[11] Another reason why banks exchange reserves on the interbank market is that not all banks borrow the reserves they need directly from the central bank but prefer to cover their needs for reserves exclusively in the interbank market.[12]

3 Stylized Facts

Until June 2010, three phases of the financial crisis can be identified. The first phase began in August 2007, when tensions in the euro area money market arose. The second phase started in September 2008 with the collapse of Lehman Brothers and the third phase began in December 2009, when the Greek debt crisis became apparent.

The three phases are reflected by the Eurosystem's lending to the euro area banking system (see the yellow line in Figure 1).

Before the crisis, the Eurosystem covered the banking sector's "actual" liquidity needs, resulting from reserve requirements and autonomous factors (blue line), so that the banking sector fulfilled its reserve requirements smoothly over a reserve maintenance period. The two lines representing the current account holdings (red line) and the reserve requirements (green line) almost coincide. This changed with the beginning of the financial crisis in August 2007. From then on, the Eurosystem allowed the credit institutions to "frontload" required reserves. At the beginning of a reserve maintenance period ample liquidity

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[11]The reason for the latter is that the Eurosystem acts as a clearing institution by operating the most important interbank payment system in the euro area which implies that payments between banks are made by exchanging deposits on their reserve accounts at the Eurosystem.

[12]In the euro area, more than 1.700 banks are eligible to participate in the MROs. However, less than 500 banks actually take part in these operations (European Central Bank, 2007, p. 89). For a respective theoretical analysis see Neyer and Wiemers (2004).
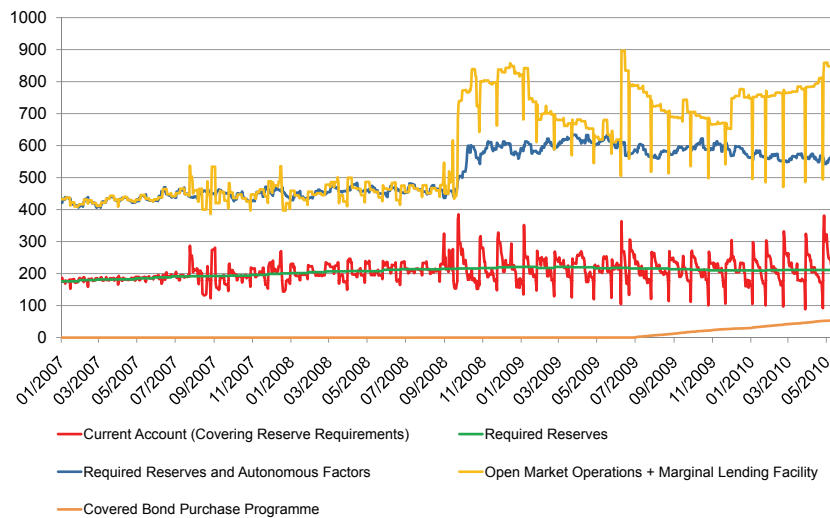
Figure 1: Needs of the Banking Sector for Reserves and Provision by the Eurosystem (EUR Billions). Data: ECB.

was provided, while over the course of the maintenance period the liquidity supply was gradually adjusted downwards so that over a maintenance period, the Eurosystem still only provided that amount of liquidity which corresponded to the banking sector's actual liquidity needs. This changed significantly with the beginning of the second phase of the financial crisis. The banking sector's demand for reserves increased strongly, exceeding by far its actual needs, and this demand was fully satisfied by the Eurosystem. By the end of 2009, the situation in the money markets improved which came along with a reduction in central bank lending. However, the uncovering of the Greek debt crisis in December 2009 led again to an increase in the banking sector's demand for reserves widening again the gap between central bank lending and the actual liquidity needs of the banking sector.[13]

Figure 2 displays the use of the Eurosystem's standing facilities. Until the beginning of the second phase of the financial crisis, they were not used intensively. However, with the beginning of the second phase banks started to place massive liquidity in the deposit facility. Also the use of the credit facility increased, especially after the collapse of Lehman Brothers and the intensification of the Greek debt crisis in May 2010. However, the use

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[13]Furthermore, the following aspects should be noted with respect to Figure 1. The increase in central bank lending in June 2009 goes in line with the first one-year maturity longer term refinancing operation conducted by the Eurosystem. This was one of its non-standard measures adopted during the crisis as well as the purchase of government bonds which started in July 2009 (see European Central Bank (2009c) for details). The frequent downward spikes of the yellow line are due to the liquidity absorbing fine tuning operations (collection of fixed term deposits) conducted by the ECB on the last day of a reserve maintenance period. And finally, the increase in the banking sector's liquidity needs with the beginning of the second phase is primarily the result of an increase in the autonomous liquidity factor "Liabilities to non-euro area residents denominated in euro" (see the Eurosystem's weekly financial statement, available at the ECB's website.)

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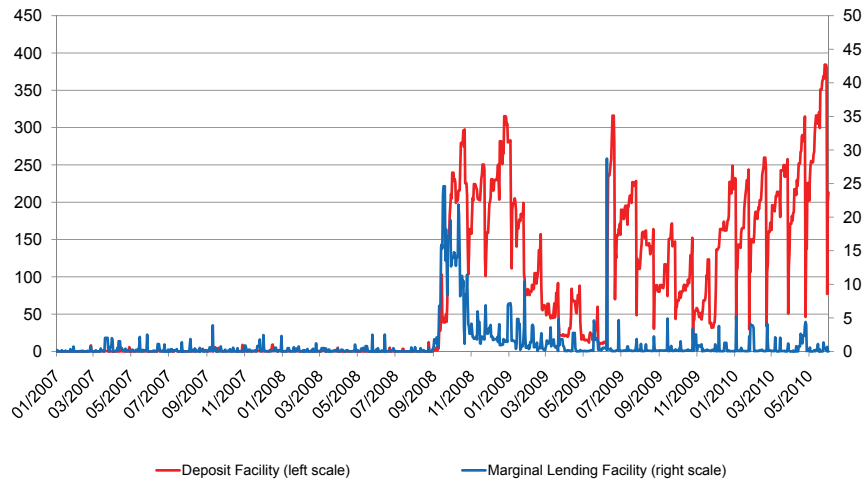
Figure 2: The Use of the Eurosystem's Facilities (EUR Billions). Data: ECB.
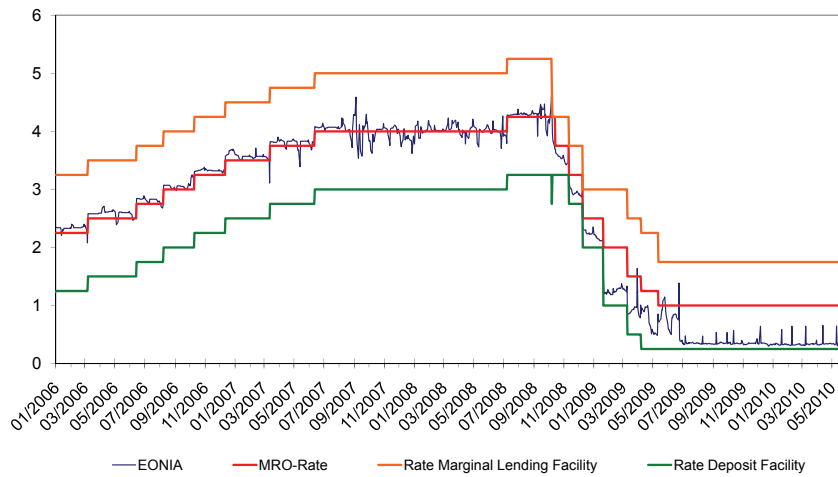
Figure 3: EONIA and Key ECB Interest Rates (Percentage). Data: ECB and Deutsche Bundesbank.

of the deposit facility exceeded by far the use of the lending facility. At the same time, transaction in the interbank market for reserves decreased significantly (European Central Bank, 2009a,b, 2010). This decrease in interbank market activities came along with a systematic fall of the EONIA[14] below the MRO-rate (see Figure 3). Note that usually, there is a positive spread between the interbank market rate and MRO-rate.[15]

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[14]EONIA is the abbreviation for *E*uro *O*vernight *I*ndex *A*verage. It is a market index computed as the weighted average of overnight unsecured lending transactions undertaken by a representative panel of banks. For more information on this reference rate see www.euribor.org.

[15]For respective empirical analyses see, for example, Nyborg, Bindseil, and Strebulaev (2002), Ayuso and Repullo (2003), Ejerskov, Moss, and Stracca (2003), Nyborg, Bindseil, and Strebulaev (2002), and Neyer and Wiemers (2004). For a theoretical explanation see, for example, Ayuso and Repullo (2003) and Neyer and Wiemers (2004).

To sum up, the second and third phase of the financial crisis were associated with (1) a strong increase in the banking sector's demand for reserves in the Eurosystem's tender procedures, (2) a strong increase in the use of the Eurosystem's deposit facility, (3) a smaller but still notable increase in the use of the marginal lending facility, (4) a decrease in transactions in the interbank market for reserves, and (5) a systematic fall of the EONIA below the MRO-rate.

4   The Model

4.1   Framework

In this section, we introduce a one-period model which replicates the main institutional features of the euro area market for reserves. There is a central bank and a large number of risk-neutral, price-taking commercial banks. Each commercial bank faces autonomous liquidity needs. The banks can obtain and place liquidity at the central bank. Furthermore, they may borrow and lend liquidity in the interbank market.

There are two ways of obtaining liquidity from the central bank. First, a commercial bank can take part in a refinancing operation (RO) and borrow the amount $K \geq 0$ at the rate i^{RO}. Second, it can use a standing credit facility and borrow the amount $CF \geq 0$ at the rate i^{CF}. Both credit operations with the central bank have to be based on adequate collateral. The costs of holding collateral are equal to $\alpha > 0$ per unit of liquidity. Consequently, borrowing K in the RO costs

$$C^{RO} = \left(i^{RO} + \alpha\right) K \tag{1}$$

and borrowing the amount CF in the credit facility costs

$$C^{CF} = \left(i^{CF} + \alpha\right) CF. \tag{2}$$

The central bank also offers a deposit facility. A commercial bank can place liquidity $DF \geq 0$ in this facility at the rate i^{DF} so that costs of the deposit facility are

$$C^{DF} = -i^{DF} DF. \tag{3}$$

The rates on the facilities form a symmetric corridor around the RO-rate. We thus have $i^{RO} = (i^{CF} + i^{DF})/2$ with $i^{DF} < i^{RO} < i^{CF}$.

The commercial banks can borrow and lend liquidity in the interbank market.[16] A bank's position in that market is B. If $B > 0$, the bank will borrow liquidity at the rate i^{IBM}. If $B < 0$, it will lend at this rate. Trading in the interbank market involves participation costs $0.5\gamma B^2$ with $\gamma > 0$.[17] Therefore, a bank's costs in the interbank market are

$$C^{IBM} = i^{IBM}B + \tfrac{1}{2}\gamma B^2. \tag{4}$$

There are two types $j = a, b$ of commercial banks which differ with respect to their autonomous liquidity needs A. Type a faces an autonomous liquidity deficit ($A^a > 0$) whereas type b has an autonomous liquidity surplus ($A^b < 0$). Half of the population of banks is of type a while the rest is of type b. In the following, we will refer to these types simply as bank a and bank b. The extent of a bank's surplus or deficit is uncertain. There are two states of the world, each occurring with probability 0.5. In state L, the deficit $A_L^a > 0$ of bank a is relatively small and the surplus $|A_L^b| > 0$ is small, too. In state H, bank a has a large deficit $A_H^a > A_L^a$ while bank b faces a large surplus $|A_H^b| > |A_L^b|$. In each state of the world, bank a's deficit is higher than bank b's surplus, so that there is always a liquidity deficit at the aggregate level. This aggregate deficit $D := A_L^a + A_L^b = A_H^a + A_H^b$, which can only be covered by the central bank, is certain. The numerical example given in Table 1 exemplifies the interrelation between the banks' autonomous liquidity needs.

	Bank a	Bank b	Total Deficit D
State L	$A_L^a = 80$	$A_L^b = -60$	20
State H	$A_H^a = 100$	$A_H^b = -80$	20

Table 1: Autonomous Liquidity Needs in the Different States of the World

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[16] As we consider a one-period model, we do not distinguish between different maturities for central bank credits or interbank market credits. Our focus on the impact of participation costs in the interbank market on banks' liquidity management when the central bank offers both, a credit and a deposit facility, allows us to explain several stylized facts observed in the euro area during the financial crisis. A discussion of the influence of different maturities on banks' liquidity management is an important issue for further research but goes beyond the scope of this paper.

[17] This quadratic form is a common approach of modeling costs in the interbank market (see, for example, Campbell, 1987; Bartolini, Bertola, and Prati, 2001). It reflects increasing marginal costs of searching for banks with matching liquidity needs and costs resulting from the need to split large transactions into many small ones to work around credit lines.

The sequence of moves is as follows: First, each bank chooses and receives the amount of liquidity K that it wishes to obtain in the central bank's RO. Thus, the central bank pursues a full allotment policy by totally satisfying each bid K of the banks. Then, the state of the world realizes, each bank learns its autonomous liquidity needs A, and chooses the amount B that it borrows or lends in the interbank market.[18] Simultaneously, it decides whether or not to use the facilities. Each bank aims at minimizing its total expected liquidity costs. We solve this optimization problem backwards. First, we determine a bank's optimal behavior in the interbank market and its optimal use of the facilities (second stage). Then, we derive the banks' optimal borrowing in the central bank's RO (first stage) and the equilibrium.

4.2 Interbank Market and Facilities (Second Stage)

After the state of the world has been realized, a bank knows its actual liquidity needs A. Then, it has to decide how much to trade in the interbank market and in how far to use the facilities offered by the central bank. Its optimization problem reads

$$C^{IBM} + C^{CF} + C^{DF} =: f(B, CF, DF) \rightarrow \min_{B, CF \geq 0, DF \geq 0} \tag{5}$$

$$\text{s. t. } K + B + CF = A + DF, \tag{6}$$

where the left hand side of (6) reflects the different liquidity sources and the right hand side reflects the liquidity uses. Solving this optimization problem, we can restrict attention to $i^{IBM} \in [i^{DF}; i^{CF} + \alpha]$. If the interbank rate were smaller than i^{DF}, no bank would be willing to supply credit as it would pay more to place excess liquidity in the deposit facility. If it were larger than $i^{CF} + \alpha$, no bank would demand credit in the interbank market. Instead, banks would use the credit facility to overcome a liquidity shortage. Denoting optimal values by the superscript *opt*, we obtain

Lemma 1: *In the second stage of the model, the optimal behaviour of a bank*

- *with a liquidity deficit $A - K > 0$ has the following properties:*

---

[18] Note that whenever it is not necessary to distinguish between the states of the world or the banks, we drop the indices from our notation.

○ if $i^{IBM} + \gamma(A - K) \leq i^{CF} + \alpha$, then

$$
\begin{aligned}
B^{opt} &= A - K, \\
CF^{opt} &= 0, \\
DF^{opt} &= 0,
\end{aligned}
$$

○ if $i^{IBM} + \gamma(A - K) > i^{CF} + \alpha$, then

$$
\begin{aligned}
B^{opt} &= \frac{i^{CF} + \alpha - i^{IBM}}{\gamma} &< A - K, \\
CF^{opt} &= A - K - \frac{i^{CF} + \alpha - i^{IBM}}{\gamma} &> 0, \\
DF^{opt} &= 0,
\end{aligned}
$$

- with a liquidity surplus $K - A \geq 0$ has the following properties:

○ if $i^{IBM} - \gamma(K - A) \geq i^{DF}$, then

$$
\begin{aligned}
B^{opt} &= -(K - A), \\
CF^{opt} &= 0, \\
DF^{opt} &= 0,
\end{aligned}
$$

○ if $i^{IBM} - \gamma(K - A) < i^{DF}$, then

$$
\begin{aligned}
B^{opt} &= -\frac{i^{IBM} - i^{DF}}{\gamma} &> -(K - A), \\
CF^{opt} &= 0, \\
DF^{opt} &= K - A - \frac{i^{IBM} - i^{DF}}{\gamma} &> 0.
\end{aligned}
$$

Proof: see appendix.

To interpret the Lemma, consider a bank which faces a liquidity deficit $A - K > 0$. As the interbank rate i^{IBM} is never below i^{DF}, the bank does not borrow in the interbank market to place the liquidity in the deposit facility, $DF^{opt} = 0$. However, by borrowing $A - K$ in the interbank market, the bank can close its deficit. Then, the marginal costs of this transaction, consisting of the interest payment and the marginal participation costs, are equal to $i^{IBM} + \gamma(A - K)$. If these marginal costs are (weakly) smaller than the marginal costs $i^{CF} + \alpha$ of the credit facility, the bank will indeed cover its total deficit in the interbank market ($B^{opt} = A - K$) without relying on the credit facility ($CF^{opt} = 0$). Otherwise, the bank will use the interbank market and/or the credit facility,

i.e. $0 \leq B^{opt} < A - K$ and $CF^{opt} > 0$. The bank will cut borrowing in the interbank market until both marginal costs are equal $(i^{IBM} + \gamma B^{opt} = i^{CF} + \alpha)$.

The interpretation of the behavior of a bank with a liquidity surplus $K - A \geq 0$ goes along the same lines. This bank does not use the credit facility and has to decide whether to place its excess liquidity in the interbank market or in the deposit facility. Obviously, the bank will lend its total surplus in the interbank market $(B^{opt} = -(K - A), DF^{opt} = 0)$ if the marginal return of this transaction net of participation costs $(i^{IBM} - \gamma (K - A))$ is not below the marginal return i^{DF} of the deposit facility. However, if $i^{IBM} - \gamma (K - A)$ falls short of i^{DF}, the bank will cut lending until its marginal net return equals i^{DF}.

Lemma 1 has implications for the transaction volume $|B|$ in the interbank market. On the one hand, a bank is willing to borrow some amount equal to $|B|$ in this market only if the marginal costs $i^{IBM} + \gamma|B|$ of borrowing do not exceed the marginal costs $i^{CF} + \alpha$ of the credit facility. On the other hand, a bank is willing to lend an amount equal to $|B|$ only if the marginal net return $i^{IBM} - \gamma|B|$ is at least as large as the marginal return i^{DF} of the deposit facility. Both conditions imply

$$|B| \leq \frac{\frac{1}{2}(i^{CF}+\alpha-i^{DF})}{\gamma} =: |B|^{marg}, \tag{7}$$

where $|B|^{marg}$ denotes the transaction volume, at which the marginality condition of the borrower as well as the lender is met with equality.

In either state of the world, bank b faces an autonomous liquidity surplus $(A^b < 0)$, so that after a possible bidding in the RO, it has excess liquidity $K^b - A^b$. Consequently, if at all, bank b will be a supplier of liquidity in the interbank market $(B^{b^{opt}} \leq 0)$. Therefore, the market clearing condition $B^{a^{opt}} + B^{b^{opt}} = 0$ requires $B^{a^{opt}} \geq 0$, and we get

Lemma 2: *The equilibrium on the interbank market has the following properties:*

(a) *If $A^a - K^a \leq 0$, then*

$$i^{IBM^*} = i^{DF} \quad and \tag{8}$$

$$|B|^* = 0. \tag{9}$$

(b) *If $A^a - K^a \in]0, K^b - A^b[$, then*

$$i^{IBM^*} = i^{DF} + \gamma|B|^* \quad and \tag{10}$$

$$|B|^* = \min\{A^a - K^a, |B|^{marg}\}. \tag{11}$$

(c) *If* $A^a - K^a > K^b - A^b$ *then*

$$i^{IBM*} = i^{CF} + \alpha - \gamma|B|^* \quad and \tag{12}$$

$$|B|^* = \min\left\{K^b - A^b, |B|^{marg}\right\}. \tag{13}$$

(d) *If* $A^a - K^a = K^b - A^b$ *then*

$$i^{IBM*} \in \left[i^{DF} + \gamma|B|^*, i^{CF} + \alpha - \gamma|B|^*\right] \quad and \tag{14}$$

$$|B|^* = \min\left\{K^b - A^b, |B|^{marg}\right\}. \tag{15}$$

Proof: see appendix.

As pointed out above, bank b will not demand credit in the interbank market. Based on this, Lemma 2 reveals that depending on the liquidity position of bank a, there are four scenarios.

In scenario (a), bank a has excess liquidity so that it does not demand credit in the interbank market either. Accordingly, there will be no transaction in that market, $|B|^* = 0$. In this scenario, the equilibrium interbank rate is equal to the marginal return i^{DF} of the deposit facility. This ensures that neither bank a nor bank b supplies or demands credit. Instead, they put their excess liquidity in the deposit facility.

In scenario (b), bank a faces a deficit $A^a - K^a$ which is smaller than the surplus $K^b - A^b$ of bank b. Consequently, there is an aggregate liquidity surplus. As long as the deficit of bank a does not exceed $|B|^{marg}$, bank a will close the deficit exclusively by borrowing in the interbank market. Otherwise, it will borrow $|B|^{marg}$ and use the credit facility in order to close the remaining deficit. Due to the aggregate liquidity surplus, lenders compete for borrowers. Therefore, the interbank market rate will be bid down until the marginal net return $i^{IBM} - \gamma|B|^*$ of lending is equal to the marginal return i^{DF} of the deposit facility and bank b will deposit some of its excess liquidity in the deposit facility.

Scenario (c) is characterized by an aggregate liquidity deficit as the deficit $A^a - K^a$ of bank a is larger than the surplus $K^b - A^b$ of bank b. If this surplus does not exceed $|B|^{marg}$, bank b will lend all of its excess liquidity to bank a, and bank a will use the credit facility to satisfy its remaining liquidity needs. If, however, $K^b - A^b$ exceeds $|B|^{marg}$, bank b will lend $|B|^{marg}$ to bank a. Then, the credit facility (by bank a) as well as the deposit facility (by bank b) will be used. In this scenario, borrowers compete for liquidity.

Consequently, the interbank rate will be bid up until the marginal costs of borrowing in the interbank market $i^{IBM} + \gamma|B|^*$ equal the marginal costs $i^{CF} + \alpha$ of the credit facility.

In the last scenario (d), there is neither an aggregate liquidity deficit nor a surplus. Then, bank b will lend either its complete excess liquidity $K^b - A^b$ or an amount equal to $|B|^{marg}$ to bank a, depending on which amount is smaller. As no bank has the "power" to bid up or down the interbank market rate to the marginal costs of its counterparty, we can only say that i^{IBM} will be somewhere between $i^{DF} + \gamma|B|^*$ and $i^{CF} + \alpha - \gamma|B|^*$.

4.3   Optimal Borrowing in the RO (First Stage) and Equilibrium

This section concentrates on the first stage of the model. At this stage, a bank chooses that amount of liquidity $K \geq 0$ it bids for in the RO, which minimizes its expected liquidity costs. Since K also determines how much liquidity a bank will borrow or lend in the interbank market and to what extent it will use the facilities (see Lemma 1), and since each state of the world occurs with probability 0.5, the decision problem of a bank reads:

$$
\begin{aligned}
C^{RO} \quad &+ \quad \tfrac{1}{2}\left[C_L^{IBM}\left(B_L^{opt}\right) + C_L^{CF}\left(CF_L^{opt}\right) + C_L^{DF}\left(DF_L^{opt}\right)\right] \\
&+ \quad \tfrac{1}{2}\left[C_H^{IBM}\left(B_H^{opt}\right) + C_H^{CF}\left(CF_H^{opt}\right) + C_H^{DF}\left(DF_H^{opt}\right)\right] =: g(K) \to \min_{K \geq 0}.
\end{aligned}
\tag{16}
$$

Solving this problem and noting that in equilibrium, each bank's bid in the RO must be optimal given that the implied interbank rate is consistent with these bids, we obtain

Proposition: *Define*

$$
\bar{\gamma} \quad := \quad \frac{\tfrac{1}{2}(i^{RO} + \alpha - i^{DF})}{E[A^a] - D},
\tag{17}
$$

$$
\bar{\bar{\gamma}} \quad := \quad \frac{\tfrac{1}{2}(i^{CF} - i^{DF})}{A_H^a - A_L^a}, \quad and
\tag{18}
$$

$$
\widehat{\gamma} \quad := \quad \frac{\tfrac{1}{2}(i^{CF} + \alpha - i^{DF})}{A_H^a - D}.
\tag{19}
$$

Suppose that $\bar{\gamma} < \widehat{\gamma}$, then, the overall equilibrium has the following properties:

(a) *If $\gamma \leq \bar{\gamma}$:*

$$
\begin{aligned}
K^{a*} &= D, & K^{b*} &= 0, \\
|B_L|^* &= A_L^a - D, & |B_H|^* &= A_H^a - D, \\
DF_L^* &= 0, & DF_H^* &= 0, \\
CF_L^* &= 0, & CF_H^* &= 0, \\
\end{aligned}
$$

$$ i^{IBM*} \in [i^{DF} + \gamma(A^a - D), i^{CF} + \alpha - \gamma(A^a - D)], $$

$$ E[i^{IBM*}] = i^{RO} + \alpha - \gamma(E[A^a] - D). $$

(b) If $\bar{\gamma} < \gamma < \bar{\bar{\gamma}}$:

$$
\begin{aligned}
K^{a*} &= E[A^a] - \frac{\frac{1}{2}(i^{RO}+\alpha-i^{DF})}{\gamma}, & K^{b*} &= 0, \\
|B_L|^* &= A_L^a - K^{a*}, & |B_H|^* &= A_H^a - K^{a*}, \\
DF_L^* &= K^{a*} - D, & DF_H^* &= K^{a*} - D, \\
CF_L^* &= 0, & CF_H^* &= 0, \\
i_L^{IBM*} &= i^{DF} + \gamma\left(A_L^a - K^{a*}\right), & i_H^{IBM*} &= i^{DF} + \gamma\left(A_H^a - K^{a*}\right).
\end{aligned}
$$

(c) If $\gamma \geq \bar{\bar{\gamma}}$:

$$
\begin{aligned}
K^{a*} &= A_L^a - \frac{\frac{1}{2}\alpha}{\gamma}, & K^{b*} &= 0, \\
|B_L|^* &= A_L^a - K^{a*}, & |B_H|^* &= |B|^{marg}, \\
DF_L^* &= K^{a*} - D, & DF_H^* &= A_H^a - D - |B|^{marg}, \\
CF_L^* &= 0, & CF_H^* &= A_H^a - \left(K^{a*} + |B|^{marg}\right), \\
i_L^{IBM*} &= i^{DF} + \gamma\left(A_L^a - K^{a*}\right), & i_H^{IBM*} &= i^{DF} + \gamma|B|^{marg}.
\end{aligned}
$$

Proof: see appendix.

The Proposition reveals that depending on the participation cost parameter γ, we can distinguish three regimes. Let us first use Figure 4 to provide an overview of the regimes given in the Proposition before discussing them in more detail. Panel (i) shows that the surplus bank b does not bid in the RO in either regime. However, bank a's borrowing in the RO differs across the three regimes. The maximum possible amount bank a can obtain in the interbank market is bank b's surplus which is equal to $A^a - D$. In regime (a), participation costs in the interbank market are that low that bank a borrows this maximum amount (panel (iii)) and covers the remaining deficit, which corresponds to the aggregate deficit D, by borrowing in the RO (panel i). Consequently, none of the facilities is used (panel (ii) and (iv)). In regime (b) higher participation costs on the interbank market imply that bank a prefers to borrow more in the RO and less in the interbank market so that bank b is is forced to use the deposit facility to some extent. In regime (c)
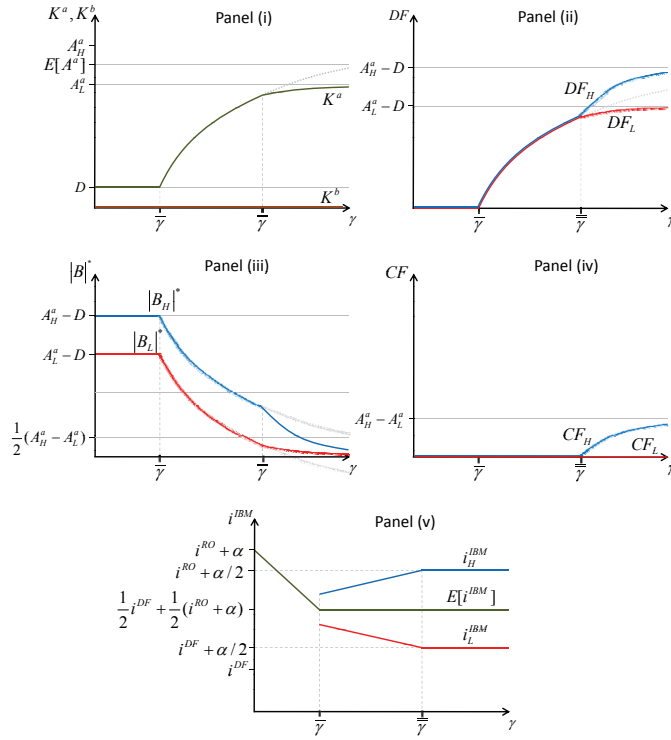
Figure 4: Borrowing in the RO, Transactions in the Interbank Market, Use of the Facilities, and the Interbank Market Rate against Participation Costs

even higher participation costs imply that bank a will even prefer to use the credit facility instead of borrowing in the interbank market if state H is realized. In the regimes (b) and (c), excess liquidity in the banking sector brings down the interbank market rate (panel (v)).

Let us now discuss the three regimes in detail. In regime (a), in which bank a borrows bank b's total surplus in the interbank market and the aggregate deficit D in the RO, there is neither an aggregate liquidity deficit nor an aggregate surplus after bank a's bidding in the RO. Therefore, there is no market power on either side of the market so that the *actual* interbank market rate will lay between its lower bound $i^{DF} + \gamma|B|^*$ and its upper bound $i^{CF} + \alpha - \gamma|B|^*$. However, the *expected* interbank rate $E[i^{IBM}]$ is determined: To ensure that bank a borrows D in the RO and relies on the interbank market afterwards, $E[i^{IBM}]$ must adjust until the marginal costs of the RO equal the expected marginal costs of borrowing bank b's surplus in the interbank market ($i^{RO} + \alpha = E[i^{IBM}] + \gamma(E[A^a] - D)$). At the same time, bank b must expect to lend its surplus instead of using the deposit facility. That is, the expected marginal net return of lending must be (weakly) higher than the marginal return of the deposit facility ($E[i^{IBM}] - \gamma(E[A^a] - D) \geq i^{DF}$). These

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two requirements for $E[i^{IBM}]$ result in $\gamma \leq \bar{\gamma}$.[19] Since in regime (a), this condition is met, bank a bids for the aggregate deficit in the RO and *expects* to cover its total remaining deficit, which corresponds to bank b's total surplus, in the interbank market. The condition $\bar{\gamma} < \hat{\gamma}$ ensures that the banks do not only *expect* to trade bank b's total surplus but also actually *do* so.[20] In this regime, an increase in γ leads to a decrease in $E[i^{IBM}]$. Intuitively, higher participation costs make the interbank market less attractive for bank a. Therefore, the expected interbank rate must decrease to offset the higher participation costs.

In regime (b) with $\bar{\gamma} < \gamma < \bar{\bar{\gamma}}$ relatively large participation costs imply that at the point $K^a = D$, bank a's marginal costs of borrowing in the RO are lower than those of using the interbank market. As a consequence, bank a expands borrowing in the RO beyond the aggregate deficit D and bank b has to use the deposit facility. For deriving the optimal K^{a*}, note that due to the aggregate liquidity surplus, the interbank rate will adjust downward until bank b's marginal net revenue of lending is equal to the marginal revenue of the deposit facility ($i^{IBM} - \gamma(A^a - K^{a*}) = i^{DF}$). Consequently, we have $E[i^{IBM}] = i^{DF} + \gamma(E[A^a] - K^{a*})$. Moreover, bank a borrows in the RO until marginal costs equal the expected marginal costs of borrowing in the interbank market ($i^{RO} + \alpha = E[i^{IBM}] + \gamma(E[A^a] - K^{a*})$). These two conditions result in the optimal K^{a*}. The implied transaction volumes $|B_L|^*$ and $|B_H|^*$ do not exceed $|B|^{marg}$. Therefore, bank a has no reason to use the credit facility. If γ increases, the interbank market will become less attractive. Then, bank a borrows more in the RO, the respective actual transaction volumes in the interbank market decrease, and bank b places more liquidity in the deposit facility. Concerning the slope of the interbank rate curve, note that a one percent increase in γ lowers the expected transaction volume $E[|B|^*]$ in the interbank market by one percent so that the marginal expected participation costs $\gamma E[|B|^*]$ are independent of γ. Accordingly, the expected interbank rate, which equals the sum of the marginal return of the deposit facility and

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[19]To see this, rearrange the first requirement to $E[i^{IBM}] = i^{RO} + \alpha - \gamma(E[A^a] - D)$ and the second requirement to $E[i^{IBM}] \geq i^{DF} + \gamma(E[A^a] - D)$ so that we must have $i^{RO} + \alpha - \gamma(E[A^a] - D) \geq i^{DF} + \gamma(E[A^a] - D)$ or $\gamma \leq \frac{\frac{1}{2}(i^{RO} + \alpha - i^{DF})}{E[A^a] - D} =: \bar{\gamma}$

[20]To see this, recall from (7) that the transaction volume in the interbank market must satisfy $|B| \leq \frac{\frac{1}{2}(i^{CF} + \alpha - i^{DF})}{\gamma}$. Therefore, Bank a borrows $A_H^a - D$ from bank b in state H (and $A_L^a - D$ in state L) only if $\gamma \leq \frac{\frac{1}{2}(i^{CF} + \alpha - i^{DF})}{A_H^a - D} =: \hat{\gamma}$. This condition is met in regime (a) as we restrict our analysis to the case $\bar{\gamma} < \hat{\gamma}$. Note that if this condition were not met, the results of regime (b) would change. Then, bank a would borrow the aggregate deficit D in the RO and rely on the credit facility to some extent in state H. That is, relative to the scenario described in the Proposition, there would be less bidding in the RO and a stronger tendency to use the credit facility. This, however, does not fit to the stylized facts observed in the euro area. Therefore, we have abstained from discussing the case $\bar{\gamma} \geq \hat{\gamma}$ in detail. We do, however, derive the equilibrium for this case formally in the proof of the proposition.

the marginal expected participation costs ($E[i^{IBM}] = i^{DF} + \gamma E[|B|^*]$), does not depend on γ, either. This, however, is not true for the respective interbank rates in the two states as Figure 4 illustrates. When γ increases by one percent, $|B|$ declines by the same absolute amount in both states. Therefore, since $|B_L|^* < |B_H|^*$, a one percent increase in γ decreases $|B_L|^*$ ($|B_H|^*$) by more (less) than one percent so that the interbank rate falls (raises) in state L (H) against γ.

In the last regime (c), in which $\gamma \geq \bar{\bar{\gamma}}$, bank a again changes its behavior. Due to the very high costs, bank a prefers to cover parts of its remaining deficit by using the credit facility in state H, in which it has high liquidity needs. Therefore, bank a borrows in the RO until its marginal costs satisfy:

$$i^{RO} + \alpha = \tfrac{1}{2}(i_L^{IBM} + \gamma(A_L^a - K^{a^*})) + \tfrac{1}{2}(i^{CF} + \alpha), \tag{20}$$

where the first term on the RHS reflects the marginal costs in the interbank market in state L and the second term reflects the marginal costs of the credit facility in state H. Like in regime (b), there is an aggregate liquidity surplus so that we have $i_L^{IBM} = i^{DF} + \gamma(A_L^a - K^{a^*})$. Together with (20), this leads to the optimal K^{a^*}.[21] In regime (c), the interest rate in the interbank market does not change in γ because a one percent increase in γ decreases the transaction volume $|B|$ in the interbank market by one percent in both states of the world. Therefore, the marginal participation costs $\gamma|B|$ and thus the interbank rate remain unchanged. What happens when participation costs tend to infinity ($\gamma \to \infty$)? Then, deficit banks do not borrow in the interbank market at all. Instead, they cover their *certain* liquidity needs in the RO ($K^{a^*} = A_L^a$) and, if necessary, the *uncertain* part of their liquidity needs by using the credit facility ($CF_H^* = A_H^a - A_L^a$).[22] It is not rational to cover uncertain liquidity needs in the RO by borrowing more than A_L^a as marginal costs $i^{RO} + \alpha$ would then be higher than the expected marginal return, i.e.

$$i^{RO} + \alpha > \tfrac{1}{2}(i^{CF} + \alpha) + \tfrac{1}{2}i^{DF}, \tag{21}$$

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[21] Relative to a scenario without a credit facility (see the respective dashed line in Figure 4), the bank will thus bid less in the RO because the credit facility makes the RO relatively less attractive. Due to the smaller amount obtained in the RO, bank a will borrow more from bank b in state L, so that there is less usage of the deposit facility in this state compared to a situation without credit facility. In state H, however, bank b puts even more liquidity in the deposit facility.

[22] The amount A_L^a equals the banks' *certain* liquidity needs because they need at least this amount of liquidity in each state of the world. The difference ($A_H^a - A_L^a$) reflects the banks' uncertain part of their liquidity needs, since this liquidity will only be needed in state H.

where the marginal return on the RHS either consists of saved credit facility costs in case the liquidity is actually needed or of interest revenues in case the additional liquidity will be placed in the deposit facility. Inequality (21) holds since the rates on the facilities form a symmetric corridor around the RO-rate, $i^{RO} = (i^{CF} + i^{DF})/2$, so that (21) becomes $i^{RO} + \alpha > i^{RO} + 0.5\alpha$. Intuitively, marginal interest costs and expected marginal interest revenues are the same due to the symmetric corridor. However, the marginal collateral costs will accrue for certain when borrowing in the RO while they will accrue only with probability 0.5 if the the bank relies on the credit facility when necessary. This argument is valid for any γ. Banks do not cover uncertain liquidity needs by borrowing in the RO, since marginal costs would then always be higher than the expected marginal return.

5   Discussion

5.1   Explanation of the Stylized Facts

Our theoretical analysis indicates that the stylized facts identified in section 3 can be explained by a strong increase of the costs of participating in the overnight interbank market during the financial crisis. Deficit banks found it more attractive to cover a higher part of their liquidity needs in the Eurosystem's refinancing operations rather than in the interbank market (regime (b)). Furthermore, some deficit banks had no access to this market since lending to them was regarded as being risky.[23] Those banks were forced to cover their total deficit by borrowing from the central bank (regime (c) for $\gamma \to \infty$). The increased demand was fully satisfied by the ECB. Therefore, transactions in the interbank market fell and the amount of outstanding central bank credits to the banking sector exceeded by far the banking sector's actual liquidity needs. Due to the excess liquidity in the banking sector, surplus banks had to place liquidity in the Eurosystem's deposit facility and the EONIA fell below the MRO-rate. Moreover, since banks do not cover uncertain liquidity needs in the central bank's refinancing operations (see page 19), blocked deficit banks used the marginal lending facility when it turned out to be necessary.

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[23]To see why lenders are unwilling to expose themselves to counterparty credit risk on the unsecured overnight interbank market, consider e.g. a bank with a liquidity surplus of 10 Million EUR and suppose that the deposit rate (marginal lending rate) of the central bank is 0.25% (1.75%), as it was the case for the ECB during the second half of 2009. The bank can either place the liquidity in the deposit facility and earn approximately 70 EUR the next day. Or the bank can lend overnight in the interbank market. Then, it will earn at most 486 EUR because the interbank market rate cannot exceed the marginal lending rate. Consequently, the bank will refuse lending in the interbank market if more that 416 EUR are at risk implying that lending must de facto be riskless. In the light of this argument, it is not surprising that empirically, an interest rate spread between the secured and unsecured overnight interbank market segment is virtually non-existent in the euro area.

Two aspects are particularly noteworthy. First, according to our model the strong use of the deposit facility is not the result of *precautionary motives*. If banks held central bank balances for precautionary reasons, they would cover *uncertain liquidity needs* by borrowing in the Eurosystem's tender procedures. However, as long as the probability of high liquidity needs is not sufficiently higher than the probability of low liquidity needs (in our model these probabilities are assumed to be the same), this behavior is not rational. Borrowing in the tender procedures and hoarding the liquidity in the deposit facility as a precaution is more expensive than using the credit facility if necessary. Consequently, we offer a further explanation for the strong use of the deposit facility, namely that surplus banks were not able to place their excess liquidity at adequate conditions in the inter-bank market since there were deficit banks preferring to borrow from the central bank rather than in the interbank market and since lending to other deficit banks would have been associated with too much credit risk.[24] The second aspect is that the Eurosystem assumed an *intermediary function* between banks as surplus banks place excess liquidity at the Eurosystem and deficit banks borrow liquidity directly from the Eurosystem. This intermediary function was reinforced by measures the Eurosystem adopted during the crisis. It narrowed the symmetric corridor that the rates on the facilities form around the MRO-rate and it reduced the requirements that collateral has to fulfill in credit operations. Both measures made transactions with the ECB relatively more attractive than interbank market transactions.[25]

5.2 Policy Implications

The ECB aims at reversing its assumed intermediary function. As pointed out, for example, by Smaghi (2008), "... a situation, in which banks simply find transactions with an intermediating ECB safer, cheaper and more convenient than trading with their peers, would be an unwelcome scenario from the point of view of the ECB ... It should be stressed

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[24]Note that in our model, there is no uncertainty about funding liquidity in the interbank market. Either a deficit bank is generally able to get liquidity in that market ($\gamma < \infty$) or it is not ($\gamma \to \infty$). However, considering this funding liquidity risk in our model, i.e. introducing a further uncertainty, would not change our result that the intensive use of the deposit facility is not due to precautionary motives. Also for this kind of uncertainty, the argument holds that as long as the probability of not getting refinanced in the interbank market is not sufficiently higher than the probability of getting refinanced, borrowing in the central bank's refinancing operations and hoarding the liquidity in the deposit facility is more expensive than using the credit facility if necessary. This may be different when explicitly considering longer-term refinancing operations offered by the central bank. However, this goes beyond the scope of this paper.

[25]In our model, the former measure is reflected by an increase in i^{DF} and a decrease in i^{CF}. The latter is reflected by a decrease in α. The proposition shows that both imply a decrease in $|B|^*$.

that our intermediation role is clearly a temporary measure which should be reversed as soon as possible...".[26] The obvious way to achieve this goal is to reduce participation costs. However, the high costs of participating in the interbank market are the result of high uncertainty about how strongly individual banks are affected by asset losses. Consequently, a reduction in participation costs cannot be achieved by central bank measures. A possibility for the Eurosystem to reactivate the interbank market is to allot only the benchmark amount in the tender procedures instead of fully satisfying total bids. However, in that case deficit banks will use the more costly credit facility instead as long as the costs of participating in the interbank market are high. A further possibility for the Eurosystem to reactivate interbank market activities is to make borrowing from and placing liquidity at the central bank less attractive. This can be achieved, for example, by tightening the criteria for eligible collateral or by expanding the corridor that the rates on the facilities form around the MRO-rate. However, also these measures will increase the banks' liquidity costs and/or may even arouse liquidity problems in the banking sector.

Consequently, as long as the costs of participating in the interbank market are high, the Eurosystem faces a trade-off. On the one hand, it aims at reactivating interbank market activities. On the other hand, it aims at supporting the general economic policy of the EU and the stability of the banking sector.[27] These objectives may require a policy leading to low liquidity costs for the banks, and they forbid a policy which may arouse liquidity problems in the banking sector. Therefore, we propose to undertake measures to reactivate the interbank market gradually over time. Over time uncertainty should decrease so that also participation costs become lower and the intermediation function becomes less important.[28]

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[26] It goes beyond the scope of this paper to discuss whether the reversion of the intermediation role assumed by the Eurosystem is generally desirable. Some authors argue that even in normal times excess reserves (i.e. in the euro area amounts held in the deposit facility) should be remunerated at the policy rate so that the opportunity costs of holding reserves are equal to zero, see, for example, Goodfriend (2002) and Borio and Disyatat (2009). In this case, there may be no need for an interbank market to distribute reserves. For a first discussion of a large intermediation by the Eurosystem also in normal times we refer the reader to Smaghi (2010).

[27] The primary objective of the Eurosystem's monetary policy is to maintain price stability and, as long as this objective is not prejudiced, to support the general economic policy of the EU (Treaty on the Functioning of the European Union, Article 127). Concerning the objective to support the stability of the banking sector, note that the Eurosystem's liquidity management shall ensure that the monetary policy transmission mechanism works properly and that in a financial crisis possible liquidity problems do not result in solvency problems (González-Páramo, 2009). The latter clarifies that during a financial crisis one objective of the Eurosystem's liquidity management is to support the stability of the banking sector.

[28] Acting as an intermediary may imply that the ECB runs the risk of supporting banks with solvency problems. This issue, which is beyond the scope of this paper, certainly deserves further research.

6 Summary

This paper theoretically explains several stylized facts concerning the euro area banks' liquidity management and its consequences after the collapse of Lehman Brothers and draws policy implications concerning the Eurosystem's liquidity management. It is shown that the stylized facts can be explained by a strong increase in participation costs on the interbank market in combination with the possibility of a nearly unlimited use of central bank credit. The increased participation costs imply that banks having a liquidity deficit prefer to cover their deficit by borrowing from the central bank rather than in the interbank market or have no access to that market anymore. This induces banks with a liquidity surplus to place their excess liquidity in the central bank's deposit facility. Thus, the central bank assumes an intermediary function between banks. The result is an aggregate liquidity surplus in the banking sector which implies a systematic fall of the EONIA below the policy rate. Concerning the implications for the Eurosystem's liquidity management we argue that as long as the interbank market does not function properly, measures to reactivate the interbank market conflict with aims from the monetary policy perspective and the financial stability perspective. Therefore, we propose to undertake these measures gradually over time.

Appendix

Proof of Lemma 1

To begin with, note that for a given B, it follows from (6) and $i^{DF} < i^{CF}$ that

$$CF = \begin{cases} A - K - B & \text{if} \quad B < A - K, \\ 0 & \text{if} \quad B \geq A - K, \end{cases} \tag{22}$$

$$DF = \begin{cases} 0 & \text{if} \quad B \leq A - K, \\ B + K - A & \text{if} \quad B > A - K. \end{cases} \tag{23}$$

Substitution of (2), (3), (4), (22) and (23) in (5) and (6) gives

$$i^{IBM} B + \tfrac{1}{2}\gamma B^2 + \begin{cases} (i^{CF} + \alpha)(A - K - B) & \text{if} \quad B \leq A - K \\ -i^{DF}(B + K - A) & \text{if} \quad B > A - K \end{cases} =: f(B) \to \min_{B}.$$

Now, we can prove the Lemma by inspecting the properties of

$$\frac{\partial f(B)}{\partial B} = \begin{cases} i^{IBM} + \gamma B - (i^{CF} + \alpha) & \text{if} \quad B < A - K, \\ i^{IBM} + \gamma B - i^{DF} & \text{if} \quad B > A - K. \end{cases} \tag{24}$$

Note that $f(B)$ is not differentiable at point $B = A - K$. Moreover, note that $\frac{\partial f(B)}{\partial B}$ is increasing in B, the limit of $\frac{\partial f(B)}{\partial B}$ as B tends to $(A - K)$ from below satisfies

$$\lim_{B \to (A-K)-} \frac{\partial f(B)}{\partial B} = i^{IBM} + \gamma(A - K) - (i^{CF} + \alpha)$$

and the limit of $\frac{\partial f(B)}{\partial B}$ as B tends to $(A - K)$ from above satisfies

$$\lim_{B \to (A-K)+} \frac{\partial f(B)}{\partial B} = i^{IBM} + \gamma(A - K) - i^{DF}.$$

Accordingly, we need to distinguish three cases:

1. If $\lim_{B \to (A-K)-} \frac{\partial f(B)}{\partial B} > 0$ (and thus $i^{IBM} + \gamma(A - K) > i^{CF} + \alpha$), it follows from (24) that the optimal transaction B^{opt} in the interbank market is defined by $i^{IBM} + \gamma B^{opt} - (i^{CF} + \alpha) = 0$, implying $B^{opt} = \frac{i^{CF} + \alpha - i^{IBM}}{\gamma}$.

2. If $\lim_{B \to (A-K)+} \frac{\partial f(B)}{\partial B} < 0$ (and thus $i^{IBM} + \gamma(A - K) < i^{DF}$), it follows from (24) that B^{opt} is defined by $i^{IBM} + \gamma B^{opt} - i^{DF} = 0$, implying $B^{opt} = -\frac{i^{IBM} - i^{DF}}{\gamma}$.

3. If $\lim_{B \to (A-K)-} \leq 0$ and $\lim_{B \to (A-K)+} \geq 0$ (and thus $i^{IBM} + \gamma(A-K) \in [i^{DF}, i^{CF} + \alpha]$), it follows from (24) that $B^{opt} = A - K$.

Restricting attention to $i^{IBM} \in [i^{DF}, i^{CF} + \alpha]$, these three cases imply

$$B^{opt} = \begin{cases} \min\left\{A - K, \frac{i^{CF} + \alpha - i^{IBM}}{\gamma}\right\} & \text{if} \quad A - K > 0, \\ -\min\left\{K - A, \frac{i^{IBM} - i^{DF}}{\gamma}\right\} & \text{if} \quad K - A \geq 0. \end{cases} \tag{25}$$

Together with (22) and (23), this leads to the optimal values stated in the Lemma. ∎

Proof of Lemma 2

We prove the Lemma by inspecting the market clearing condition $B^{a^{opt}} + B^{b^{opt}} = 0$. Restricting attention to $i^{IBM} \in [i^{DF}, i^{CF} + \alpha]$ and $K^b - A^b > 0$, we proceed in two steps:

1. Suppose $K^a - A^a \geq 0$. Then, substitution of (25) in $B^{a^{opt}} + B^{b^{opt}} = 0$ gives

$$-\min\left\{K^a - A^a, \frac{i^{IBM} - i^{DF}}{\gamma}\right\} - \min\left\{K^b - A^b, \frac{i^{IBM} - i^{DF}}{\gamma}\right\} = 0.$$

This condition is met only if $i^{IBM} = i^{DF}$; substitution of this in (25) gives $|B^{opt}| = 0$. Denoting these values by i^{IBM*} and $|B|^*$, we obtain (8) and (9).

2. Suppose $A^a - K^a > 0$. Then, substitution of (25) in $B^{a^{opt}} + B^{b^{opt}} = 0$ gives

$$\min\left\{A^a - K^a, \frac{i^{CF} + \alpha - i^{IBM}}{\gamma}\right\} - \min\left\{K^b - A^b, \frac{i^{IBM} - i^{DF}}{\gamma}\right\} = 0. \tag{26}$$

Now, we can distinguish three subcases:

a. Suppose $A^a - K^a \in]0, K^b - A^b[$. Then, if $i^{IBM} > i^{DF} + \gamma(A^a - K^a)$ and thus $\min\left\{K^b - A^b, \frac{i^{IBM} - i^{DF}}{\gamma}\right\} > A^a - K^a$, the LHS of (26) is negative. Hence, we can restrict attention to $i^{IBM} \leq i^{DF} + \gamma(A^a - K^a)$ so that (26) becomes

$$\min\left\{A^a - K^a, \frac{i^{CF} + \alpha - i^{IBM}}{\gamma}\right\} - \frac{i^{IBM} - i^{DF}}{\gamma} = 0, \tag{27}$$

which is met if $i^{IBM} = i^{DF} + \gamma \min\left\{A^a - K^a, |B|^{marg}\right\}$; substitution of this in (25) gives $|B^{opt}| = \min\left\{A^a - K^a, |B|^{marg}\right\}$, see (10) and (11). As the LHS of (27) is strictly decreasing in i^{IBM}, this equilibrium is unique.

b. Suppose $A^a - K^a > K^b - A^b$. Then, if $i^{IBM} < i^{CF} + \alpha - \gamma(K^b - A^b)$ and thus $\min\left\{A^a - K^a, \frac{i^{CF}+\alpha-i^{IBM}}{\gamma}\right\} > K^b - A^b$, the LHS of (26) is positive. Hence, we can restrict attention to $i^{IBM} \geq i^{CF} + \alpha - \gamma(K^b - A^b)$ so that (26) becomes

$$\frac{i^{CF}+\alpha-i^{IBM}}{\gamma} - \min\left\{K^b - A^b, \frac{i^{IBM}-i^{DF}}{\gamma}\right\} = 0, \qquad (28)$$

which is met if $i^{IBM} = i^{CF} + \alpha - \gamma \min\left\{K^b - A^b, |B|^{marg}\right\}$; substitution of this in (25) gives $|B^{opt}| = \min\left\{K^b - A^b, |B|^{marg}\right\}$, see (12) and (13). As the LHS of (28) is strictly decreasing in i^{IBM}, this equilibrium is unique.

c. Suppose $A^a - K^a = K^b - A^b$. By parallel arguments as above, (26) implies $i^{IBM} \in [i^{DF} + \gamma \min\left\{K^b - A^b, |B|^{marg}\right\}, i^{CF} + \alpha - \gamma \min\left\{K^b - A^b, |B|^{marg}\right\}]$; substitution of this in (25) gives $|B^{opt}| = \min\left\{K^b - A^b, |B|^{marg}\right\}$, see (14) and (15). ■

Proof of the Proposition

We prove the Proposition in two steps. We first investigate the bidding incentives of each individual bank in the RO. Then, we derive the overall equilibrium. For an individual bank, substitution of (1), (2), (3) and (4) in (16), together with Lemma 1, gives

$$
\begin{aligned}
g(K) \;=\; & \left(i^{RO} + \alpha\right) K + \tfrac{1}{2}\left[i_L^{IBM} B_L^{opt} + \tfrac{1}{2}\gamma (B_L^{opt})^2\right] + \tfrac{1}{2}\left[i_H^{IBM} B_H^{opt} + \tfrac{1}{2}\gamma (B_H^{opt})^2\right] \\
& + \begin{cases}
\left[\begin{array}{l} \tfrac{1}{2}\left(i^{CF}+\alpha\right)\left(A_L - K - B_L^{opt}\right) \\ +\tfrac{1}{2}\left(i^{CF}+\alpha\right)\left(A_H - K - B_H^{opt}\right) \end{array}\right] & \text{if } K < A_L, \\[2em]
\left[\begin{array}{l} -\tfrac{1}{2}i^{DF}\left(B_L^{opt} + K - A_L\right) \\ +\tfrac{1}{2}\left(i^{CF}+\alpha\right)\left(A_H - K - B_H^{opt}\right) \end{array}\right] & \text{if } K \in [A_L, A_H[, \\[2em]
\left[\begin{array}{l} -\tfrac{1}{2}i^{DF}\left(B_L^{opt} + K - A_L\right) \\ -\tfrac{1}{2}i^{DF}\left(B_H^{opt} + K - A_H\right) \end{array}\right] & \text{if } K \geq A_H,
\end{cases}
\end{aligned}
$$

and thus

$$
\begin{aligned}
\frac{\partial g(K)}{\partial K} = \ & i^{RO} + \alpha + \tfrac{1}{2}\left(i_L^{IBM} + \gamma B_L^{opt}\right)\frac{\partial B_L^{opt}}{\partial K} + \tfrac{1}{2}\left(i_H^{IBM} + \gamma B_H^{opt}\right)\frac{\partial B_H^{opt}}{\partial K} \\
& - \begin{cases}
\begin{bmatrix} \tfrac{1}{2}(i^{CF} + \alpha)\left(1 + \frac{\partial B_L^{opt}}{\partial K}\right) \\[4pt] + \tfrac{1}{2}(i^{CF} + \alpha)\left(1 + \frac{\partial B_H^{opt}}{\partial K}\right) \end{bmatrix} & \text{if} \quad K < A_L, \\[20pt]
\begin{bmatrix} \tfrac{1}{2}i^{DF}\left(1 + \frac{\partial B_L^{opt}}{\partial K}\right) \\[4pt] + \tfrac{1}{2}(i^{CF} + \alpha)\left(1 + \frac{\partial B_H^{opt}}{\partial K}\right) \end{bmatrix} & \text{if} \quad K \in [A_L, A_H[, \\[20pt]
\begin{bmatrix} \tfrac{1}{2}i^{DF}\left(1 + \frac{\partial B_L^{opt}}{\partial K}\right) + \tfrac{1}{2}i^{DF}\left(1 + \frac{\partial B_H^{opt}}{\partial K}\right) \end{bmatrix} & \text{if} \quad K \geq A_H.
\end{cases}
\end{aligned} \tag{29}
$$

For a given liquidity need A_i with $i = L, H$, it follows from (25) that

$$
\frac{\partial B_i^{opt}}{\partial K} = \begin{cases}
-1 & \text{if} \quad i_i^{IBM} + \gamma(A_i - K) \leq i^{CF} + \alpha \quad \text{and} \quad K < A_i, \\
0 & \text{if} \quad i_i^{IBM} + \gamma(A_i - K) > i^{CF} + \alpha \quad \text{and} \quad K < A_i,
\end{cases} \tag{30}
$$

$$
\frac{\partial B_i^{opt}}{\partial K} = \begin{cases}
0 & \text{if} \quad i_i^{IBM} + \gamma(A_i - K) < i^{DF} \quad \text{and} \quad K \geq A_i, \\
-1 & \text{if} \quad i_i^{IBM} + \gamma(A_i - K) \geq i^{DF} \quad \text{and} \quad K \geq A_i.
\end{cases} \tag{31}
$$

Substitution of (25), (30) and (31) in (29) gives

$$
\begin{aligned}
\frac{\partial g(K)}{\partial K} = \ & i^{RO} + \alpha \\
& - \begin{cases}
\begin{bmatrix} \tfrac{1}{2}\min\left\{i_L^{IBM} + \gamma(A_L - K), i^{CF} + \alpha\right\} \\[4pt] + \tfrac{1}{2}\min\left\{i_H^{IBM} + \gamma(A_H - K), i^{CF} + \alpha\right\} \end{bmatrix} & \text{if} \quad K < A_L, \\[20pt]
\begin{bmatrix} \tfrac{1}{2}\max\left\{i_L^{IBM} + \gamma(A_L - K), i^{DF}\right\} \\[4pt] + \tfrac{1}{2}\min\left\{i_H^{IBM} + \gamma(A_H - K), i^{CF} + \alpha\right\} \end{bmatrix} & \text{if} \quad K \in [A_L, A_H[, \\[20pt]
\begin{bmatrix} \tfrac{1}{2}\max\left\{i_L^{IBM} + \gamma(A_L - K), i^{DF}\right\} \\[4pt] + \tfrac{1}{2}\max\left\{i_H^{IBM} + \gamma(A_H - K), i^{DF}\right\} \end{bmatrix} & \text{if} \quad K \geq A_H.
\end{cases}
\end{aligned} \tag{32}
$$

As $\frac{\partial g(K)}{\partial K}$ is (weakly) increasing in K, we can already derive three preliminary results:

1. An equilibrium with $K^{a^*} \geq A_L^a$ would be feasible only if $\frac{\partial g(K^a)}{\partial K^a}\big|_{A_L^a} \leq 0$. Due to (32), this condition requires

$$
i^{RO} + \alpha \leq \tfrac{1}{2}i_L^{IBM} + \tfrac{1}{2}\min\left\{i_H^{IBM} + \gamma(A_H^a - A_L^a), i^{CF} + \alpha\right\}. \tag{33}
$$

27

Now, note that $i^{RO} = \frac{i^{CF} + i^{DF}}{2}$ and that in the case of $K^{a^*} \geq A_L^a$, (8) implies $i_L^{IBM^*} = i^{DF}$. Therefore, (33) becomes $\frac{1}{2}i^{CF} + \alpha \leq \frac{1}{2}\min\left\{i_H^{IBM} + \gamma(A_H^a - A_L^a), i^{CF} + \alpha\right\}$, and we can conclude that there is no equilibrium with $K^{a^*} \geq A_L^a$.

2. An equilibrium with $K^{a^*} < A_L^a$ and $K^{b^*} > 0$ would be feasible only if $\frac{\partial g(K^a)}{\partial K^a}\big|_{K^{a^*} < A_L^a} \geq 0$ and $\frac{\partial g(K^b)}{\partial K^b}\big|_{K^{b^*} > 0} = 0$. Due to (32), these conditions require

$$
\begin{aligned}
i^{RO} + \alpha \;\geq\; & \tfrac{1}{2}\min\left\{i_L^{IBM} + \gamma(A_L^a - K^{a^*}), i^{CF} + \alpha\right\} && (34) \\
+ \; & \tfrac{1}{2}\min\left\{i_H^{IBM} + \gamma(A_H^a - K^{a^*}), i^{CF} + \alpha\right\}, \\
i^{RO} + \alpha \;=\; & \tfrac{1}{2}\max\left\{i_L^{IBM} + \gamma(A_L^b - K^{b^*}), i^{DF}\right\} && (35) \\
+ \; & \tfrac{1}{2}\max\left\{i_H^{IBM} + \gamma(A_H^b - K^{b^*}), i^{DF}\right\}.
\end{aligned}
$$

(34) and (35) are met simultaneously only if the RHS of (35) is not smaller than the RHS of (34). However, since $A_H^b < A_L^b < 0$, the RHS of (35) is equal to i^{DF} for $i_L^{IBM} = i_H^{IBM} = i^{DF}$ and strictly smaller than $\frac{1}{2}(i_L^{IBM} + i_H^{IBM})$ otherwise. Moreover, since $K^{a^*} < A_L^a < A_H^a$, the RHS of (34) is equal to $i^{CF} + \alpha$ for $i_L^{IBM} = i_H^{IBM} = i^{CF} + \alpha$ and strictly larger than $\frac{1}{2}(i_L^{IBM} + i_H^{IBM})$ otherwise. Therefore, (34) and (35) cannot be met simultaneously and we can conclude that there is no equilibrium with $K^{a^*} < A_L^a$ and $K^{b^*} > 0$.

3. An equilibrium with $K^{a^*} < D := A_L^a + A_L^b = A_H^a + A_H^b$ and $K^{b^*} = 0$ would be feasible only if $\frac{\partial g(K^a)}{\partial K^a}\big|_{K^{a^*} < D} \geq 0$. Due to (32), this condition requires

$$
\begin{aligned}
i^{RO} + \alpha \;\geq\; & \tfrac{1}{2}\min\left\{i_L^{IBM} + \gamma(A_L^a - K^{a^*}), i^{CF} + \alpha\right\} && (36) \\
+ \; & \tfrac{1}{2}\min\left\{i_H^{IBM} + \gamma(A_H^a - K^{a^*}), i^{CF} + \alpha\right\}.
\end{aligned}
$$

Now, note that if $K^{a^*} < D$ and $K^{b^*} = 0$, it follows from (12) and (13) that $i_L^{IBM^*} > i^{CF} + \alpha - \gamma(A_L^a - K^{a^*})$ and $i_H^{IBM^*} > i^{CF} + \alpha - \gamma(A_H^a - K^{a^*})$. Subsitution of this in (36) yields $i^{RO} + \alpha \geq i^{CF} + \alpha$ so that we can conclude that there is no equilibrium with $K^{a^*} < D$ and $K^{b^*} = 0$.

The three preliminary results imply that we can restrict attention to equilibria with $K^{a^*} \in [D, A_L^a[$ and $K^{b^*} = 0$, which are feasible only if $\frac{\partial g(K^a)}{\partial K^a} \mid_{K^{a^*} \in [D, A_L^a[} = 0$ and $\frac{\partial g(K^b)}{\partial K^b} \mid_{0 > A_L^b} \geq 0$. Due to (32), these conditions require

$$i^{RO} + \alpha = \tfrac{1}{2} \min \left\{ i_L^{IBM} + \gamma(A_L^a - K^{a^*}), i^{CF} + \alpha \right\} \tag{37}$$
$$+ \tfrac{1}{2} \min \left\{ i_H^{IBM} + \gamma(A_H^a - K^{a^*}), i^{CF} + \alpha \right\},$$

$$i^{RO} + \alpha \geq \tfrac{1}{2} \max \left\{ i_L^{IBM} + \gamma A_L^b, i^{DF} \right\} + \tfrac{1}{2} \max \left\{ i_H^{IBM} + \gamma A_H^b, i^{DF} \right\}. \tag{38}$$

Now, it is useful to distinguish two cases:

1. Suppose $K^{a^*} = D$ and $K^{b^*} = 0$. Then, since $A_L^b = D - A_L^a$ and $A_H^b = D - A_H^a$, (37) and (38) can be rewritten to

$$i^{RO} + \alpha = \tfrac{1}{2} \min \left\{ i_L^{IBM} + \gamma(A_L^a - D), i^{CF} + \alpha \right\} \tag{39}$$
$$+ \tfrac{1}{2} \min \left\{ i_H^{IBM} + \gamma(A_H^a - D), i^{CF} + \alpha \right\},$$

$$i^{RO} + \alpha \geq \tfrac{1}{2} \max \left\{ i_L^{IBM} + \gamma(D - A_L^a), i^{DF} \right\} \tag{40}$$
$$+ \tfrac{1}{2} \max \left\{ i_H^{IBM} + \gamma(D - A_H^a), i^{DF} \right\}.$$

Since $A_H^a > A_L^a > D$, the RHS of (39) is larger than the RHS of (40). Accordingly, we can restrict attention to (39). Now, we can consider two subcases:

a. Suppose $A_H^a - D \leq |B|^{marg}$ (and thus $\gamma \leq \frac{\frac{1}{2}(i^{CF} + \alpha - i^{DF})}{A_H^a - D} =: \widehat{\gamma}$). Then, it follows from (14) and (15) that

$$i_L^{IBM^*} \in [i^{DF} + \gamma(A_L^a - D), i^{CF} + \alpha - \gamma(A_L^a - D)], \tag{41}$$

$$i_H^{IBM^*} \in [i^{DF} + \gamma(A_H^a - D), i^{CF} + \alpha - \gamma(A_H^a - D)], \tag{42}$$

so that we can rewrite (39) to $i^{RO} + \alpha = \tfrac{1}{2}[i_L^{IBM} + \gamma(A_L^a - D)] + \tfrac{1}{2}[i_H^{IBM} + \gamma(A_H^a - D)]$, implying $E[i^{IBM}] := \tfrac{1}{2}i_L^{IBM} + \tfrac{1}{2}i_H^{IBM} = i^{RO} + \alpha - \gamma(E[A^a] - D)$. This is consistent with (41) and (42) only if $i^{RO} + \alpha - \gamma(E[A^a] - D) \geq i^{DF} + \gamma(E[A^a] - D)$, implying $\gamma \leq \frac{\frac{1}{2}(i^{RO} + \alpha - i^{DF})}{E[A^a] - D} =: \bar{\gamma}$. To sum up this subcase, $K^{a^*} = D$ and $K^{b^*} = 0$ is an equilibrium if $\gamma \leq \min \{\widehat{\gamma}, \bar{\gamma}\}$.

b. Suppose $A_H^a - D > |B|^{marg} \geq A_L^a - D$ (and thus $\gamma \in]\widehat{\gamma}, \frac{\frac{1}{2}(i^{CF} + \alpha - i^{DF})}{A_L^a - D}]$). Then, it follows from (14) and (15) that

$$i_L^{IBM^*} \in [i^{DF} + \gamma(A_L^a - D), i^{CF} + \alpha - \gamma(A_L^a - D)], \tag{43}$$

$$i_H^{IBM^*} = \tfrac{1}{2}(i^{CF} + \alpha + i^{DF}), \tag{44}$$

so that we can rewrite (39) to $i^{RO} + \alpha = \frac{1}{2}[i_L^{IBM} + \gamma(A_L^a - D)] + \frac{1}{2}(i^{CF} + \alpha)$, implying $i_L^{IBM} = i^{DF} + \alpha - \gamma(A_L^a - D)$. This is consistent with (43) only if $i^{DF} + \alpha - \gamma(A_L^a - D) \geq i^{DF} + \gamma(A_L^a - D)$ implying $\gamma \leq \frac{\frac{1}{2}\alpha}{A_L^a - D} =: \widetilde{\gamma}$. To sum up this subcase, $K^{a^*} = D$ and $K^{b^*} = 0$ is an equilibrium if $\gamma \in]\widehat{\gamma}, \widetilde{\gamma}]$.

2. Suppose $K^{a^*} \in]D, A_L^a[$ and $K^{b^*} = 0$. Then, it follows from (10) and (11) that

$$i_L^{IBM^*} = i^{DF} + \gamma \min\left\{A_L^a - K^{a^*}, |B|^{marg}\right\}, \tag{45}$$

$$i_H^{IBM^*} = i^{DF} + \gamma \min\left\{A_H^a - K^{a^*}, |B|^{marg}\right\}. \tag{46}$$

Substitution of (45) and (46) in (37) and (38) yields

$$i^{RO} + \alpha = \tfrac{1}{2}\min\left\{i^{DF} + 2\gamma(A_L^a - K^{a^*}), i^{CF} + \alpha\right\} \tag{47}$$
$$+ \tfrac{1}{2}\min\left\{i^{DF} + 2\gamma(A_H^a - K^{a^*}), i^{CF} + \alpha\right\},$$

$$i^{RO} + \alpha \geq i^{DF}. \tag{48}$$

As (48) is met, we can restrict attention to (47), which can be rearranged to

$$K^{a^*} = \begin{cases} E[A^a] - \dfrac{\frac{1}{2}(i^{RO} + \alpha - i^{DF})}{\gamma} & \text{if } K^{a^*} \geq A_H^a - \dfrac{\frac{1}{2}(i^{CF} + \alpha - i^{DF})}{\gamma}, \\ A_L^a - \dfrac{\frac{1}{2}\alpha}{\gamma} & \text{if } K^{a^*} < A_H^a - \dfrac{\frac{1}{2}(i^{CF} + \alpha - i^{DF})}{\gamma}, \end{cases}$$

implying

$$K^{a^*} = \begin{cases} E[A^a] - \dfrac{\frac{1}{2}(i^{RO} + \alpha - i^{DF})}{\gamma} & \text{if } \gamma \leq \dfrac{\frac{1}{2}(i^{CF} - i^{DF})}{A_H^a - A_L^a} =: \overline{\overline{\gamma}}, \\ A_L^a - \dfrac{\frac{1}{2}\alpha}{\gamma} & \text{if } \gamma > \dfrac{\frac{1}{2}(i^{CF} - i^{DF})}{A_H^a - A_L^a} =: \overline{\overline{\gamma}}. \end{cases}$$

Due to the requirement $K^{a^*} > D$, we can sum up this subcase by stating that there is an equilibrium with $K^{b^*} = 0$ and

$$K^{a^*} = \begin{cases} E[A^a] - \frac{\frac{1}{2}(i^{RO}+\alpha-i^{DF})}{\gamma} & \text{if} \quad \gamma \in]\bar{\gamma}, \bar{\bar{\gamma}}], \\ A_L^a - \frac{\frac{1}{2}\alpha}{\gamma} & \text{if} \quad \gamma > \max\{\bar{\bar{\gamma}}, \tilde{\gamma}\}. \end{cases} \tag{49}$$

Note that only if $\bar{\gamma} < \hat{\gamma}$, we have $\bar{\gamma} < \bar{\bar{\gamma}}$, $\tilde{\gamma} < \bar{\bar{\gamma}}$ and $\tilde{\gamma} < \hat{\gamma}$. Therefore, we obtain:

- If $\bar{\gamma} < \hat{\gamma}$, there is an equilibrium with $K^{b^*} = 0$ and

$$K^{a^*} = \begin{cases} D & \text{if} \quad \gamma \leq \bar{\gamma}, \\ E[A^a] - \frac{\frac{1}{2}(i^{RO}+\alpha-i^{DF})}{\gamma} & \text{if} \quad \gamma \in]\bar{\gamma}, \bar{\bar{\gamma}}], \\ A_L^a - \frac{\frac{1}{2}\alpha}{\gamma} & \text{if} \quad \gamma > \bar{\bar{\gamma}}. \end{cases} \tag{50}$$

Substitution of (50) and $K^{b^*} = 0$ in Lemma 1 and 2 then results in the proposition.

- If $\bar{\gamma} \geq \hat{\gamma}$, there is an equilibrium (not included in the proposition) with $K^{b^*} = 0$ and

$$K^{a^*} = \begin{cases} D & \text{if} \quad \gamma \leq \tilde{\gamma}, \\ A_L^a - \frac{\frac{1}{2}\alpha}{\gamma} & \text{if} \quad \gamma > \tilde{\gamma}. \end{cases} \qquad \blacksquare$$

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